Covering Arrays, Set Covers, Algorithms and their Complexity

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Generation of Covering Arrays for Abstract Combinatorial Test Suites

Covering Arrays for Combinatorial Testing
- Covering Arrays (CAs) provide the theoretical means for Combinatorial Testing (CT).
- Columns of a CA map to the parameters of a system under test.
- Rows of a CA encode the individual test cases.
- Their combinatorial properties guarantee that derived test sets cover all $t$-way interactions.
- To apply CT to arbitrary SUTs, we need to be able to generate arbitrary CAs.

Covering Arrays via Set Covers

Optimal Covering Arrays as Minimal Set Covers
- The Set Cover Problem is a well studied problem in theoretical CS.
- For a given universe $U$ and a set of blocks $S$, i.e. subsets of $U$, we want to find a minimal subset of $S$ that covers $U$.
- The CA generation problem can be interpreted as a Set Cover problem:
  - $U := T_\tau$, the set of all $t$-way interactions
  - $S := \sum_{v_i} n_i$ set of potential rows
- Then a minimal set cover represents an optimal CA

CA instance $\mathcal{S}$ instance

Construct an optimal CA(N;2,3,2), i.e. minimal N.

Algorithm for Covering Arrays via Set Covers

- This connection allows to apply Set Cover (SC) Algorithms for CA generation.
- Some existing algorithms for CA generation can be identified as classical SC algorithms applied to CA instances.

Covering Arrays and Computational Complexity

- Formulation of CA-related problems as formal complexity problems.
- Establish connections between these problems:
  - For arbitrary but fixed $t$ and $v$, it holds that
    - $\text{decSizeOMCA}(N; t, k, \{v_1, \ldots, v_k\}) \leq \text{genOMCA}_{t,v}(N; t, k, \{v_1, \ldots, v_k\})$
  - $\text{genOMCA}_{t,v}(N; t, k, \{v_1, \ldots, v_k\}) \leq \text{detSizeOMCA}(N; t, k, \{v_1, \ldots, v_k\})$
- Analyse state of the art of complexity problems related to CAs.
- Correction of statements and clarification of misinterpretation.
- The computational complexity of the Covering Array generation problem remains unknown.

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