



# In-Parameter-Order strategies for covering perfect hash families



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## ABSTRACT

Combinatorial testing makes it possible to test large systems effectively while maintaining certain coverage guarantees. At the same time, the construction of optimized covering arrays (CAs) with a large number of columns is a challenging task. Heuristic and Meta-heuristic approaches often become inefficient when applied to large instances, as the computation of the quality for new moves or solutions during the search becomes too slow. Recently, the generation of covering perfect hash families (CPHFs) has led to vast improvements to the state of the art for many different instances of covering arrays. CPHFs can be considered a compact form of a specific family of covering arrays. Their compact representation makes it possible to apply heuristic methods for instances with a much larger number of columns. In this work, we adapt the ideas of the well-known In-Parameter-Order (IPO) strategy for covering array generation to efficiently construct CPHFs, and therefore implicitly covering arrays. We design a way to realize the concept of vertical extension steps in the context of CPHFs and discuss how a horizontal extension can be implemented in an efficient manner. Further, we develop a horizontal extension strategy for CPHFs with subspace restrictions that identifies candidate columns greedily based on conditional expectation. Then using a local optimization strategy, a candidate may be adjoined to the solution or may replace one of the existing columns. An extensive set of computational results yields many significant improvements on the sizes of the smallest known covering arrays.

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## 1. Introduction

Covering Arrays (CAs) are a combinatorial design class of combinatorial designs that can be considered a generalization of orthogonal arrays. A (uniform)  $CA(N; t, k, \nu)$  is an  $N \times k$  array in which each entry is from a  $\nu$ -ary alphabet and is defined by the property that for every possible selection of  $t$  columns, every  $t$ -tuple  $\{0, 1, \dots, \nu - 1\}^t$  appears in at least one row of the sub-array. Whenever such a tuple appears in a row, we consider it *covered*, a CA can therefore also be defined as an array that covers all possible  $t$ -tuples in all  $t$ -selections of columns, further referred to as *column selections*.

The construction of CAs with a minimal number of rows is of particular interest, especially for practical applications. Given a strength  $t$ , number of columns  $k$  and alphabet of cardinality  $\nu$ , we consider the problem of generating a  $CA(N; t, k, \nu)$  as a CA instance and aim to construct a CA with the smallest number of rows possible. The smallest value of  $N$  for which

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